Two Vacuum Vignettes

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Outline

- Vacuum Decay & Lorentz Invariance.

Dvali (2011): summing "subleading" final states contributes ∞ factor to total rate.
Metastable vacua don't exist.

We study these final states and find that they have extra exponential suppressions.
No infinity of the proposed type; metastability still ok.
M. Dine, P. D., C. Park (2012)

- "Stimulated" Vacuum Decay.

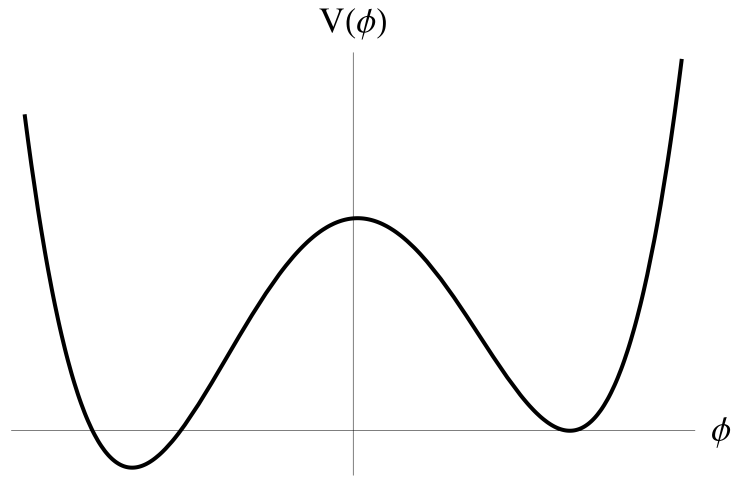
If the SM is good to 10^{12} GeV, EW vacuum is metastable. Tunneling rate is tiny.

Instead tunneling through the barrier, can we go over it?

No. But still fun to think about.

Some initial states lead to "decay," but are impossible to fabricate.
N. Arkani-Hamed, Y. Bai, P. D. (ongoing)

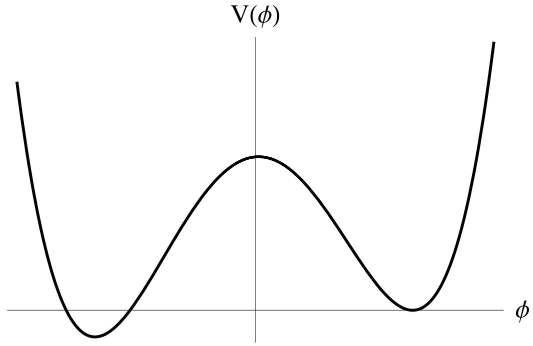
Vacuum Decay & Lorentz Invariance



$$V(\phi) = \frac{\lambda}{8}(\phi^2 - \frac{\mu^2}{\lambda})^2 + \frac{\epsilon}{2a}(\phi - a) \ , \qquad a^2 \equiv \mu^2/\lambda$$

1974: Voloshin, Kobzarev, Okun considered decay of metastable vacua in field theory. Ansatz of thin-wall bubble nucleation.

VKO assumed decay could produce bubbles at rest or in motion. Therefore, total rate included an integration over Lorentz boosts.



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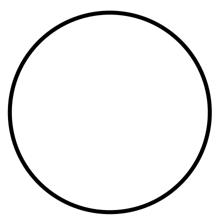
1977: Coleman addressed the problem more systematically.
Derived that leading contrib to Γ/V is via nucleation of a critical bubble



$$P^\mu = (0,0,0,0)$$

$R_0 = 3S_1/\epsilon$ (critical bubble radius)

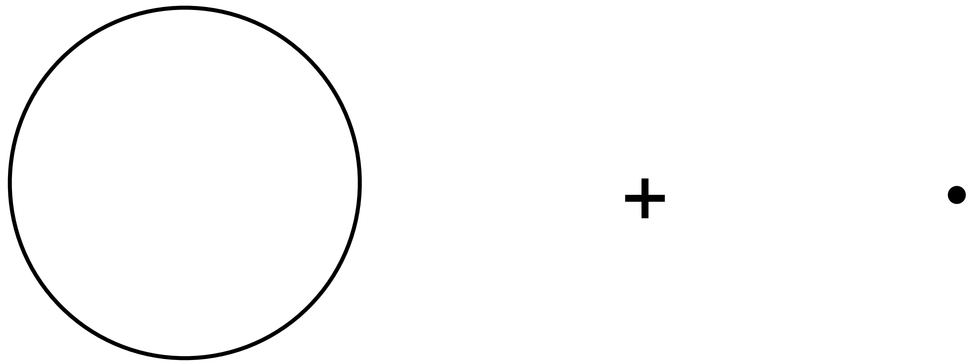
$S_1 = \mu^3/\lambda$ (wall mass/unit area)

thin-wall limit: $R_0 >> \mu^{-1} \Rightarrow \epsilon << \mu^4$

From Lorentz invariance of the *critical bubble* and translation invariance of the false vacuum, Coleman concluded that Γ/V is Poincare invariant.
Boost integration would overcount final states.

[∃ range of parameters s. th. thin-wall ok, ignoring gravity ok.]

2011: Dvali considers subleading contributions from larger bubbles + particles



$$P^\mu = (-M, 0, 0, 0) \qquad\qquad P^\mu = (M, 0, 0, 0)$$

⇒ infinite family of inequivalent, zero-energy configurations ("Dvaliballs"?)

related by boosts and labeled by

$$(\Lambda P_{\rm rel})^\mu = (2\gamma M, 0, 0, 2\gamma\beta M)$$

From Lorentz invariance *of the false vacuum,* concluded that decay rate is the same into all these configurations, integration over boosts divergent

Concluded Metastable Minkowski vacua don't exist.

Two clear problems with Dvali's argument.

(1) Euclidean computation of QM corrections is finite. Callan & Coleman 1977

No qualitative change from leading semiclassical result.

Failure of Euclidean result would be surprising.

Two clear problems with Dvali's argument.

(II) Poincare algebra: false vacuum is not Lorentz invariant.
(States invariant under space transl but not time transl are not boost invariant.)

$$-P_3 M_{03}|0\rangle = [M_{03}, P_3]|0\rangle = H|0\rangle \neq 0$$

No reason to expect rates into boosted final states to be the same.

No reason to expect a divergence proportional to Vol(SO(3,1)).

Euclidean computation

Pro: powerful, “easy”

Con: obscures contribution from individual final states,
obscures boost dependence.

Can we understand features of a Minkowski space analysis?
Suppression of rates into highly-boosted final states?

Minkowski Space Problem

First, look at the amplitude to nucleate critical bubbles + fluctuations.
Focus on thin-wall case where there is a clean separation of scales.

Perturbed bubble:

$$\phi = \phi_{\rm cl} + \delta\phi$$

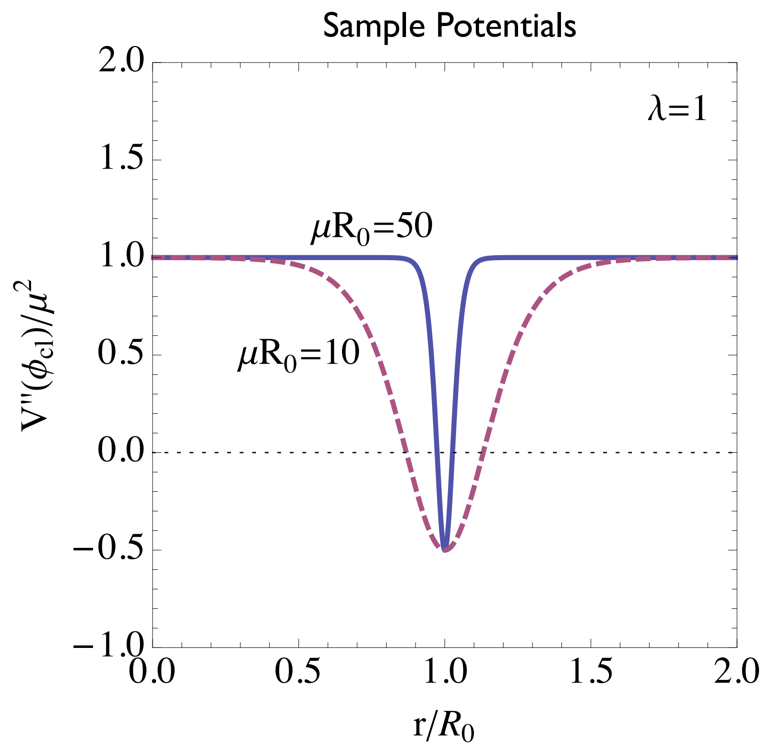
$$\phi_{\rm cl} \approx \phi_{\rm cl}(r - R(t)), \quad R(t) = \sqrt{R_0^2 + t^2}$$

Fluctuation EOM:

$$\Box\delta\phi + U(\phi_{\rm cl})\delta\phi = 0$$

$$U(\phi_{\rm cl}) \equiv V''(\phi_{\rm cl})$$

Potential variation localized near R_0



$R_0 = 3S_1/\epsilon$ (critical bubble radius)

$S_1 = \mu^3/\lambda$ (wall mass/unit area)

thin-wall limit: $R_0 \gg \mu^{-1} \Rightarrow \epsilon \ll \mu^4$

Minkowski Space Problem

Simplest fluctuations describe ripples in the bubble wall:

$$\delta\phi = \sum_{lm} \Delta_{lm} Y_{lm} \partial_r \phi_{\rm cl}$$

For early times $t \ll R_0$, unperturbed bubble is approx static. $\dot{R} = t/R$

Fluctuation EOM: $$\boxed{\ddot{\Delta}_{lm} = -\omega_{lm}^2 \Delta_{lm}, \qquad \omega_{lm}^2 = \frac{-2 + \ell(\ell+1)}{R_0^2}}$$

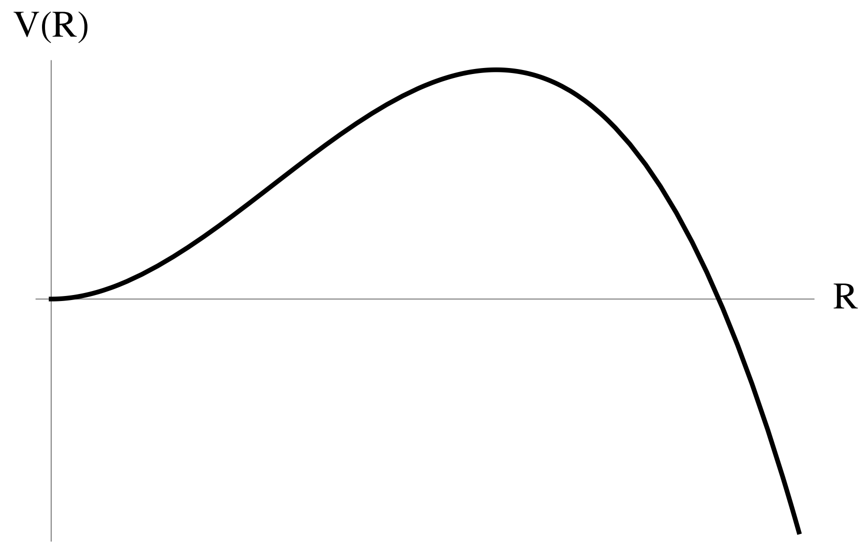
$\ell = 0$ (dilatation) mode unstable: increasing radius lowers energy
$\ell = 1$ (translation) modes are zero modes

low-ℓ 'slow' modes not really 'oscillators'; bubble grows on $1/\omega$ timescale

high-ℓ 'fast' modes well-described: oscillate many times before bubble moves

Effective Degrees of Freedom:

bubble radius R with potential



bubble translations (ignore; already considered by Coleman)

harmonic oscillators associated with higher-ℓ ripples in the wall, $\omega \sim \ell / R$

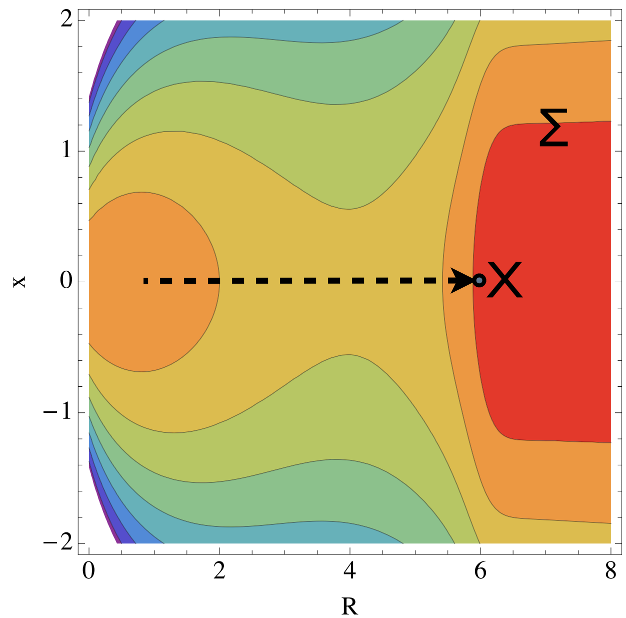
higher discretuum (internal wall structure) and continuum (particle modes), frequencies of order $\mu >> 1/R$

What is the tunneling amplitude from inside the well to outside + excited oscillator states?

Multi-dimensional tunneling: WKB exponent $\int ds\sqrt{2V}$ obtained by integrating along path of least resistance

Banks, Bender, Wu 1973

In pure semiclassical limit, only tunnel to one final state X on the zero-energy surface Σ



Others states on Σ can only be reached asymptotically by going through X

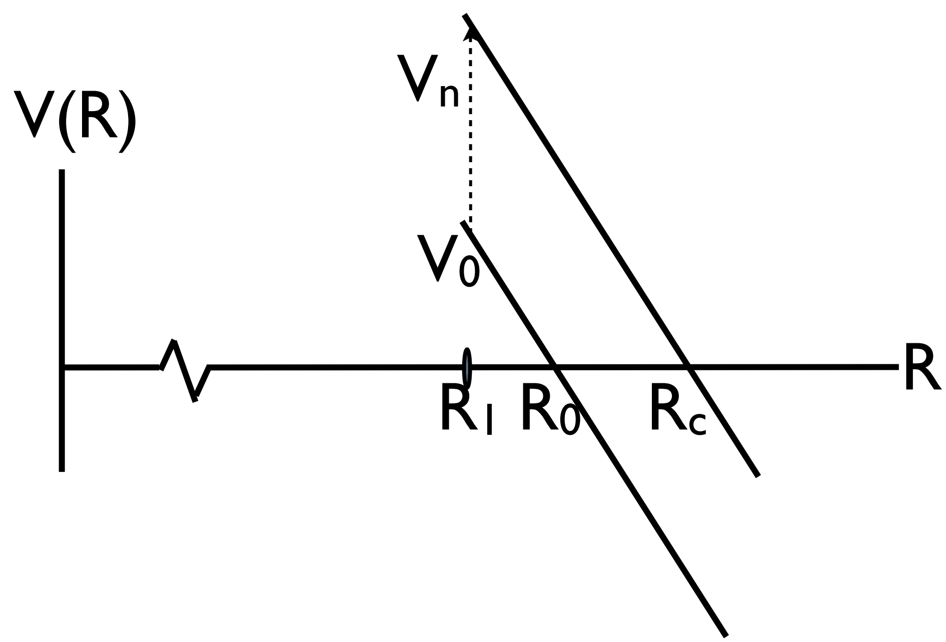
Next order in perturbation theory:
scatter and excite modes on the way out.

$$\mathcal{H} \supset \left(\omega \frac{\partial \omega}{\partial R}\right)\bigg|_{R_1} \delta R\ \phi^2$$

Imagine scattering happens at some radius R_1 into a mode $n\omega \sim n\ell/R_1$

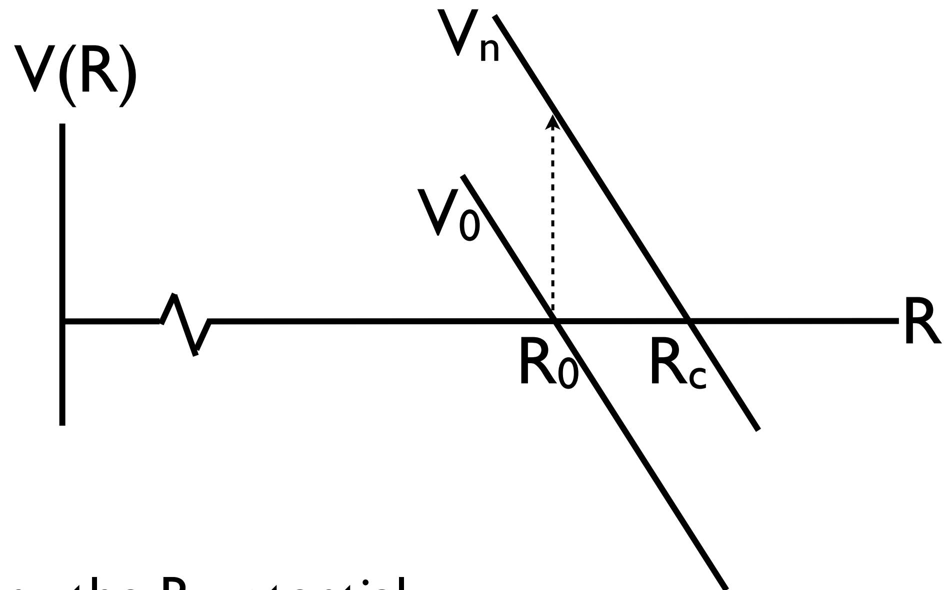
In the adiabatic approximation, oscillator creates an effective potential for R

$$V_n(R;n) \simeq V_0(R) + n\omega(R_1) \simeq V_0 + n\ell/R_1$$



Then R tunnels the rest of the way out through V_n .

Dominant path:
(1) tunnel for a long time with no excitation
(2) near $R_1=R_0$, make perturbative jump(s)
(3) tunnel again



Price for excitation is tunneling further along the R potential.

$\Rightarrow$ amplitude suppressed at R_c beyond SO(3,1)-symmetric amplitude,

additional WKB factor of order

$$\boxed{\mathcal{F} \sim \exp\left(-\delta R\sqrt{2MV_n(R_0)}\right)}$$

$$\delta R = \frac{n\omega(R_0)}{V_0'(R_0)}\,, \quad M \simeq S_1 R_0^2\,, \quad V_n(R_0) \simeq n\omega\,, \quad V_0'(R_0) \simeq S_1 R_0$$

$$\Rightarrow \quad \boxed{\mathcal{F} \sim \exp\left[-((n\omega)^3/S_1)^{1/2}\right]}$$

Check adiabatic approximation:
given ω, get δR, induces shift $\delta\omega$ in ω. Small?

$$\frac{\delta\omega}{\omega} \sim \frac{1}{\omega}\delta R \frac{\partial\omega}{\partial R}\bigg|_{R_0} \sim \frac{\delta R}{R_0} \sim \frac{n\omega}{\mu}(\mu R_0)^{-2}$$

small by definition in thin-wall

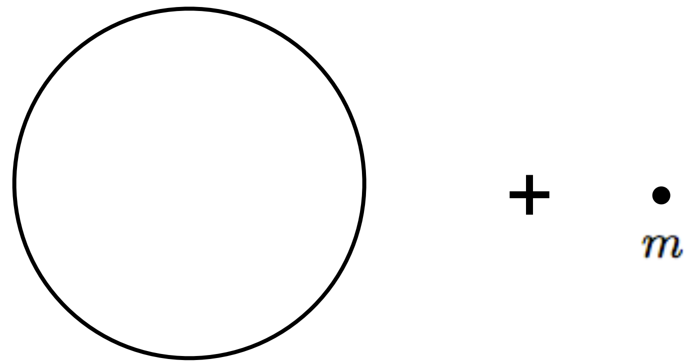
So we can probe excitations well into the continuum if excitation happens in a single jump,

or further if via series-of-jumps (still exponential suppression, more small perturbative factors.)

Total rate is Lorentz-invariant and high-energy contributions are very suppressed.

Back to Lorentz properties & Dvali's proposal:

particle at rest + supercritical bubble



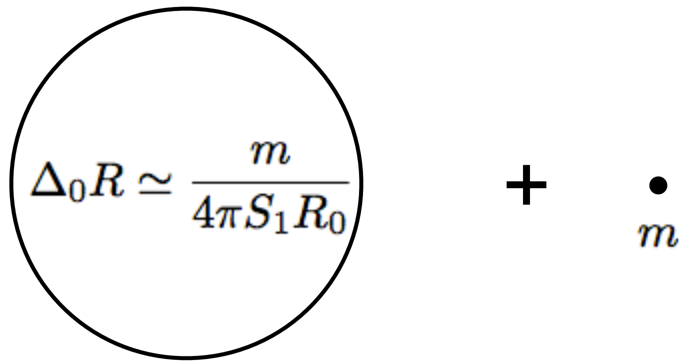
$$R_i \equiv R_0 + \Delta_0 R$$
$$R_0 = 3S_1/\epsilon \quad \dot{R}(0) = 0$$

Bubble Energy: $E = \dfrac{4\pi S_1 R^2}{\sqrt{1-\dot{R}^2}} - \dfrac{4\pi}{3}\epsilon R^3$, so $\boxed{E \simeq -4\pi S_1 R_0 \Delta_0 R\,, \quad \Delta_0 R \simeq \dfrac{m}{4\pi S_1 R_0}}$

Bubble Lagrangian: $L = -4\pi S_1 R^2 \sqrt{1-\dot{R}^2} + \dfrac{4\pi}{3}\epsilon R^3$

EOM: $\ddot{R} = \dfrac{\epsilon}{S_1}(1-\dot{R}^2)^{3/2} - \dfrac{2}{R}(1-\dot{R}^2)$

Evolution: $R(t) \approx R_i + \dfrac{1}{2}\left(\dfrac{\epsilon}{S_1} - \dfrac{2}{R_i}\right)t^2$



Boosts will make particle more energetic. What happens to the t=0 bubble?

Embedding: $R(t)^2 - x^2 - y^2 - z^2 = 0$ $\qquad R(t) \approx R_i + \frac{1}{2}\left(\frac{\epsilon}{S_1} - \frac{2}{R_i}\right)t^2$

Leading order in boost parameter:

$$R_i^2 - \left(1 - 3\beta^2\left(\frac{R_i}{R_0} - 1\right)\right)x^2 - y^2 - z^2 = 0\,.$$

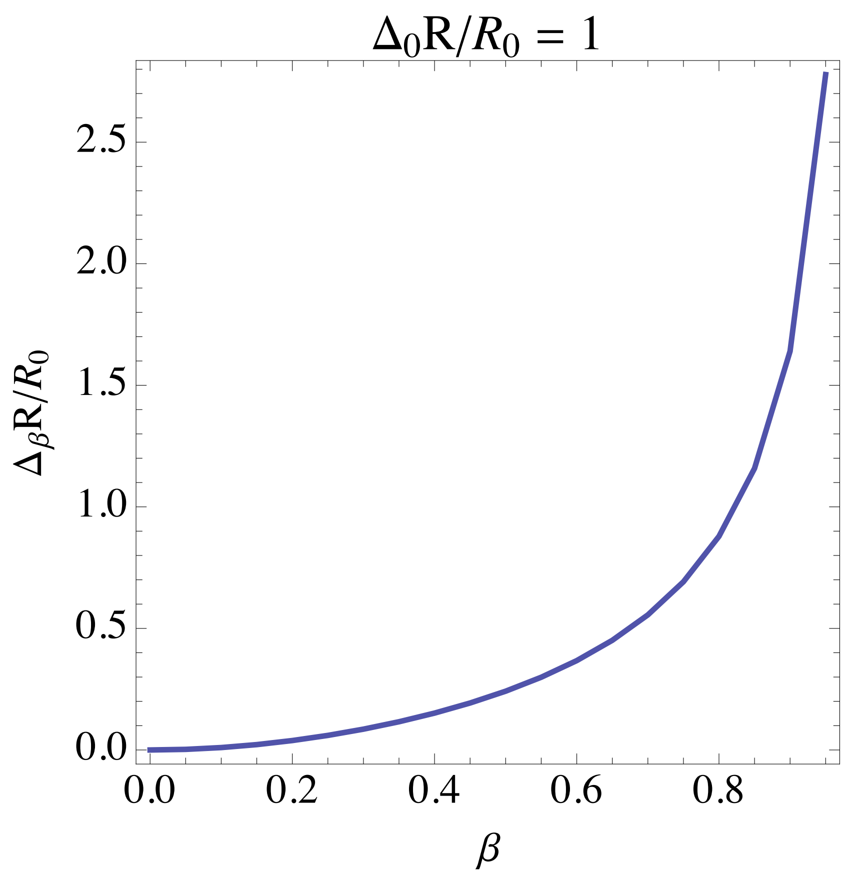
Undeformed in $R_i \to R_0$ limit

Radius **increases**: $\Delta R = \Delta_0 R + \Delta_\beta R$ $\qquad \Delta_\beta R \simeq \frac{1}{2}\beta^2 \Delta_0 R$

Contrast with a rigid sphere, which would length-contract.
Supercritical bubbles must grow under boosts to conserve energy.

Beyond leading order, can solve for radius of boosted bubble numerically

$\Delta_\beta R \simeq \frac{1}{2}\beta^2 \Delta_0 R$ becomes



Boost also deforms bubbles:
dumps energy into higher-ℓ modes

We conclude that the partial rates are highly suppressed with β.

Summary of part one

False vacua are not Lorentz-invariant

In subleading decay processes,
nucleate particles/excitations + non-Lorentz-invariant bubbles

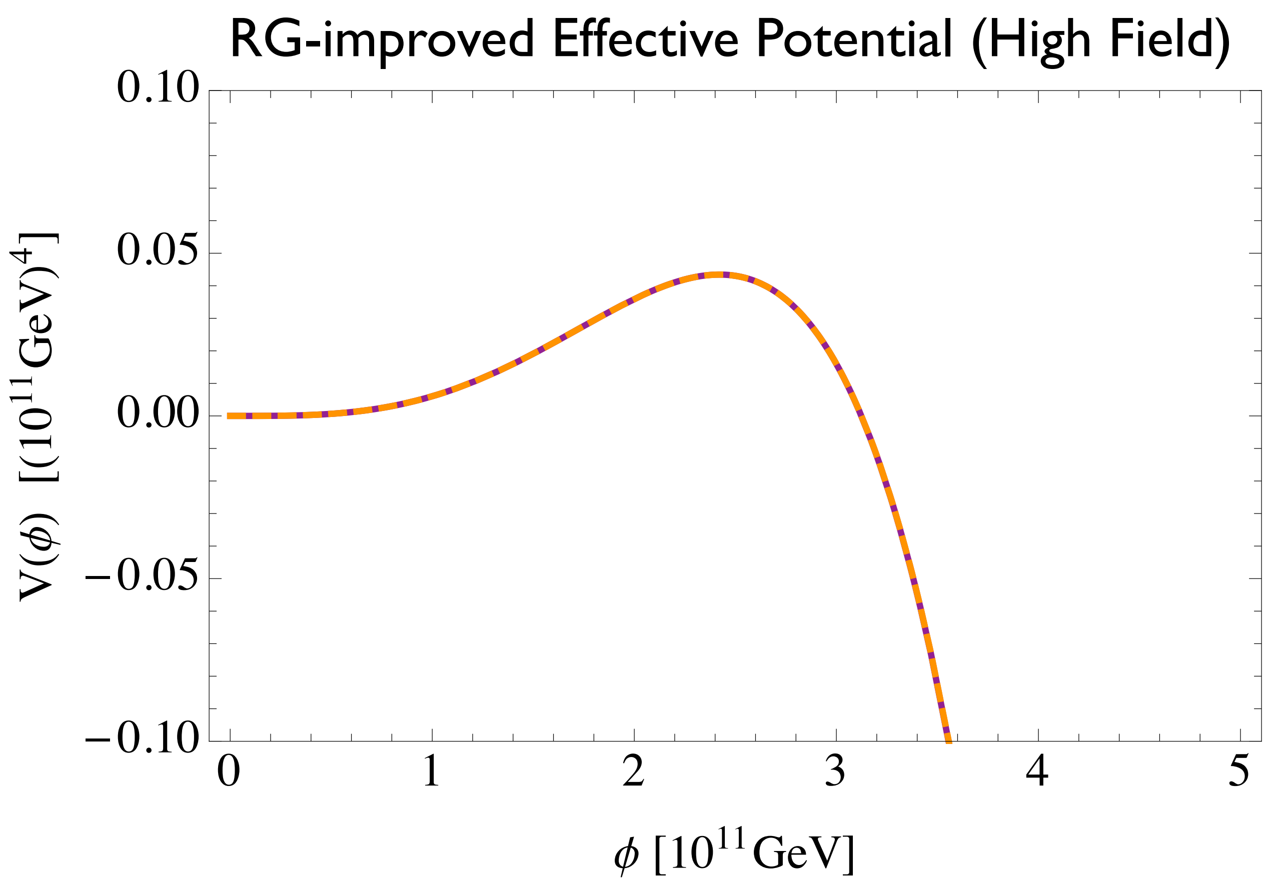
Production of high-energy excitations is exponentially suppressed because of larger nucleated bubble radius

Lorentz boosts also increase bubble radius $\Rightarrow$ exponential suppression

“Stimulated” Vacuum Decay

This part really will be just for fun.

“Stimulated” Vacuum Decay



Adapted from Degrassi, Di Vita, Elias-Miro, Espinosa, Giudice, Isidori, Strumia 2012

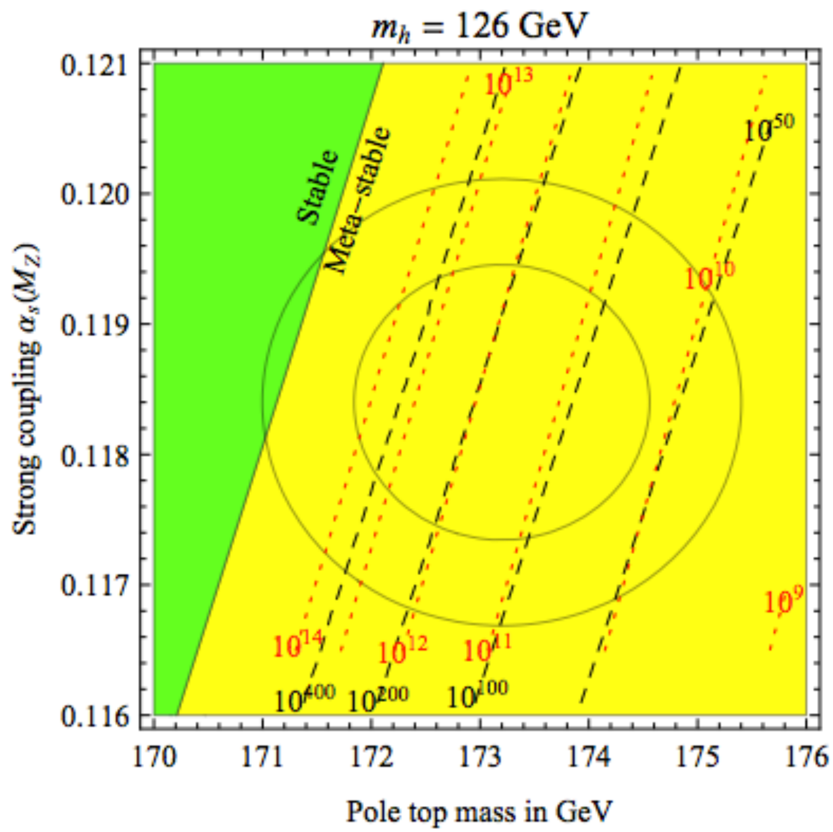
Higgs potential turns over near 10^{11} GeV and is unbounded from below. New physics must stabilize it!

Two possibilities:

(1) new physics is below the instability scale.
Our vacuum probably stable.

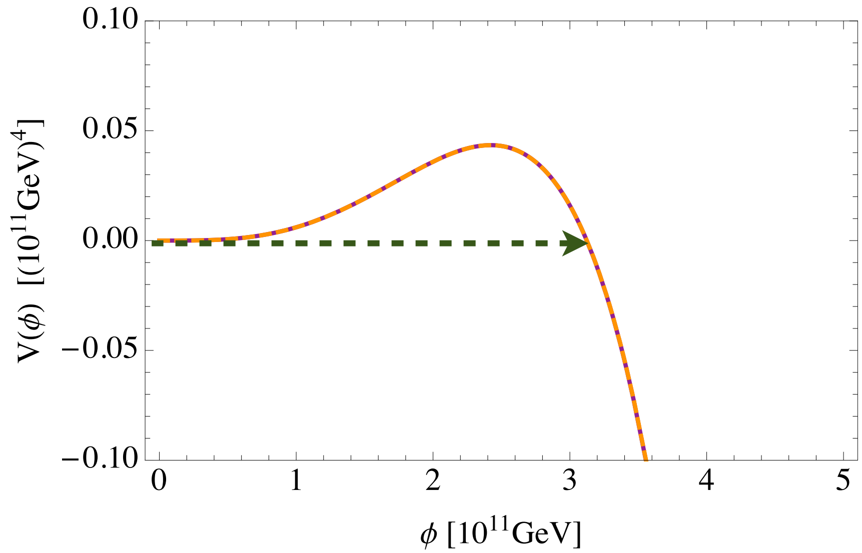
Two possibilities:

(II) new physics is only above instability scale.
Our vacuum metastable, lifetime ~ 10^{100} y



Degrassi, Di Vita, Elias-Miro,
Espinosa, Giudice, Isidori,
Strumia 2012

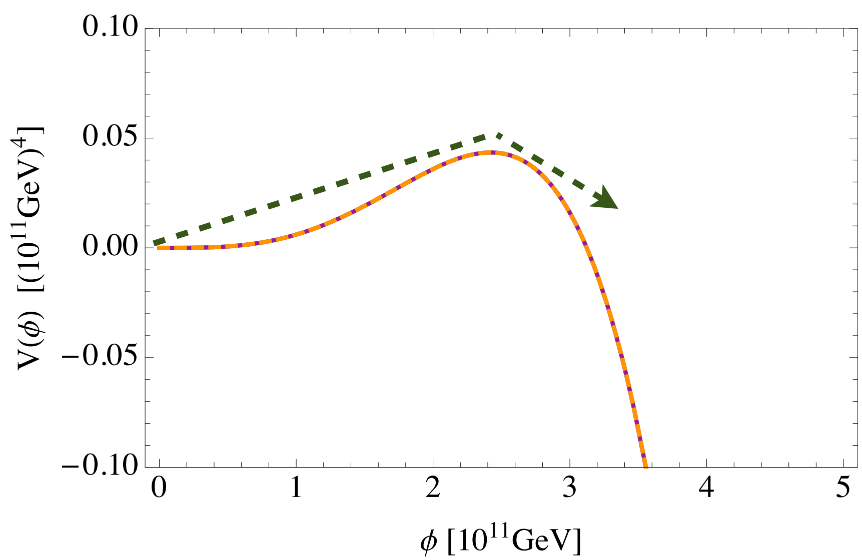
$10^{100} = \infty.$



Are there processes similar to vacuum decay that are not exponentially unlikely?

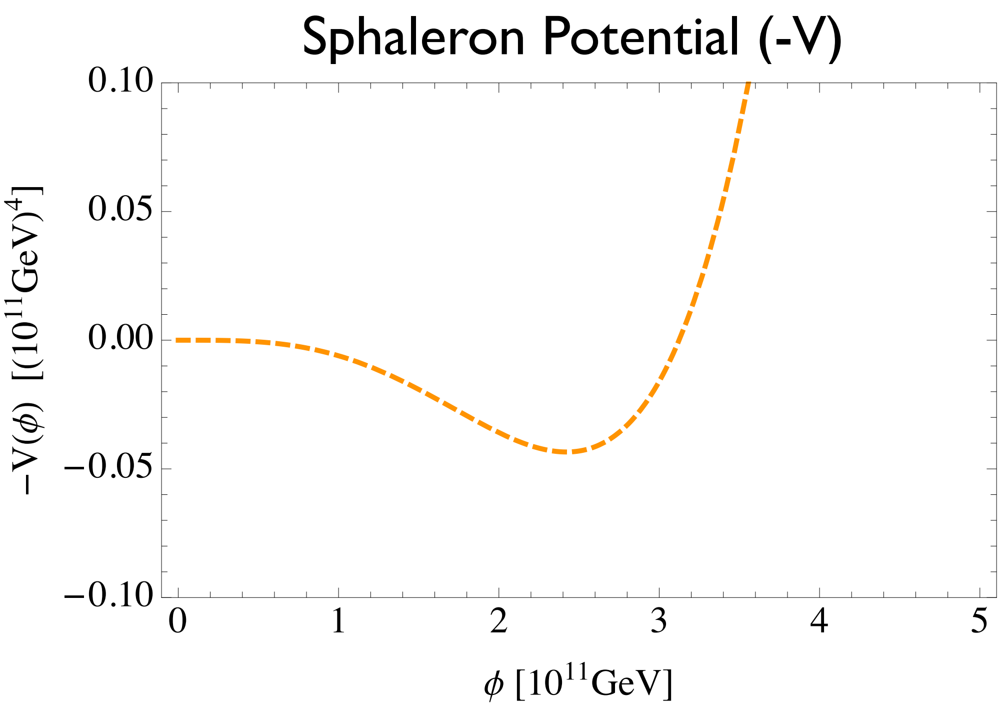
For example,
going *over* the hill?

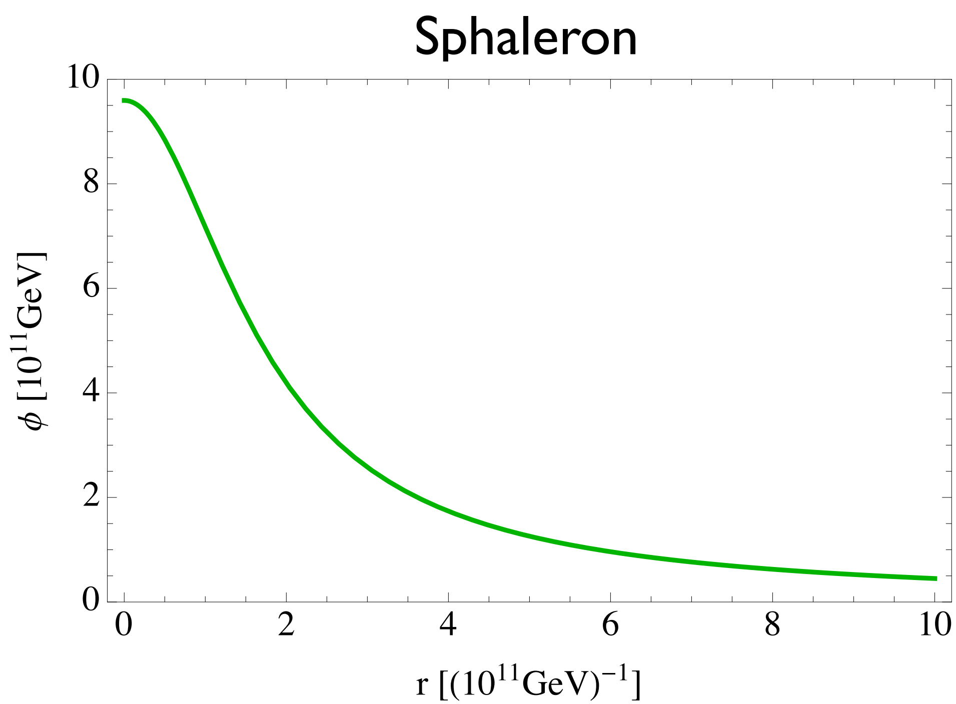
Rubakov & Son (1994)



Such a process would, of course, destroy the universe.

The potential admits a static, unstable, spherically-symmetric critical bubble ('sphaleron')





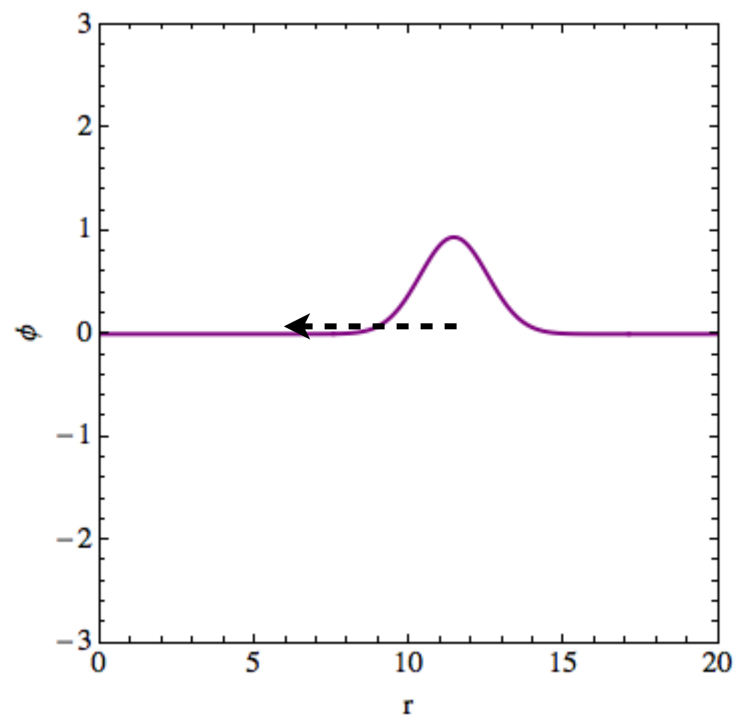
$$\frac{\partial V}{\partial \phi} = \partial_{rr}\phi + \frac{2}{r}\partial_r \phi$$

$$E_{sph} \approx 250 \times 10^{11} \text{ GeV}$$

If we can achieve a similar field configuration by a dynamical process, may go over the saddle point.

Imagine a classical spherical Higgs wavepacket, collapsing towards the origin

$$\langle\phi\rangle \sim A \exp\left(-\frac{(r-r_0)^2}{2\sigma^2}\right) ,$$
$$\langle\pi\rangle \sim A\sigma^{-2}(r-r_0) \exp\left(-\frac{(r-r_0)^2}{2\sigma^2}\right) .$$



Trying to mock up the sphaleron ⇒ natural value of σ is O(sphaleron width).

What is a classical Higgs wavepacket?

The fact that we're interested in classical field theory suggests a coherent state formulation at the quantum level.

If field is linear at early times,
can interpret initial coherent state in terms of avg. # particles with avg energies.

$$|f\rangle \equiv \beta \exp\left(\int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_p}} f(p) a_p^\dagger\right)|0\rangle\,, \qquad \beta \equiv \exp\left(-\bar{N}/2\right) \qquad \bar{N} = \int \frac{d^3p}{(2\pi)^3}\frac{1}{2E_p}|f(p)|^2$$

Field initial conditions fixed by
$$\langle\phi\rangle = \mathrm{Re}\ \mathfrak{F}(f/E_p)$$
$$\langle\pi\rangle = \mathrm{Im}\ \mathfrak{F}(f)$$

Then for large particle numbers the evolution of the state can be studied semiclassically.

Higgs classical potential is stable -- need top quark loops to see instability.

$\Rightarrow$ we should be asking about the quantum equation of motion

Technically, for initial value problems, should use quantum EOM from CTP formalism.

However, at 2-derivative, 1-loop level, CTP result is equivalent to

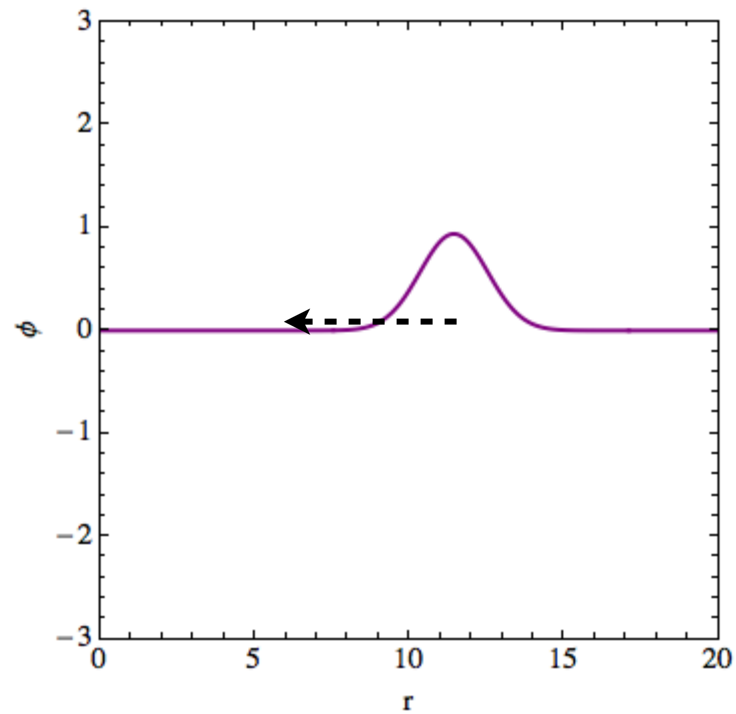
$$\Box\phi + \frac{dV}{d\phi} = 0 \longrightarrow \Box\phi + \frac{dV_{\text{eff}}}{d\phi} = 0$$

where V_{eff} is the RG-improved CW effective potential.

(With all these approximations, life becomes easy! Vanilla classical scalar field theory.)

Returning to our wavepacket,

$$\langle\phi\rangle \sim A \exp\left(-\frac{(r-r_0)^2}{2\sigma^2}\right) ,$$
$$\langle\pi\rangle \sim A\sigma^{-2}(r-r_0) \exp\left(-\frac{(r-r_0)^2}{2\sigma^2}\right) .$$



Avg particle energy, total energy, and # of particles in coherent state given by

$$p_0 \sim \sigma^{-1} , \qquad E_{tot} \sim 4\pi r_0^2 \sigma p_0^2 A^2 , \qquad N \sim E_{tot}/p_0 .$$

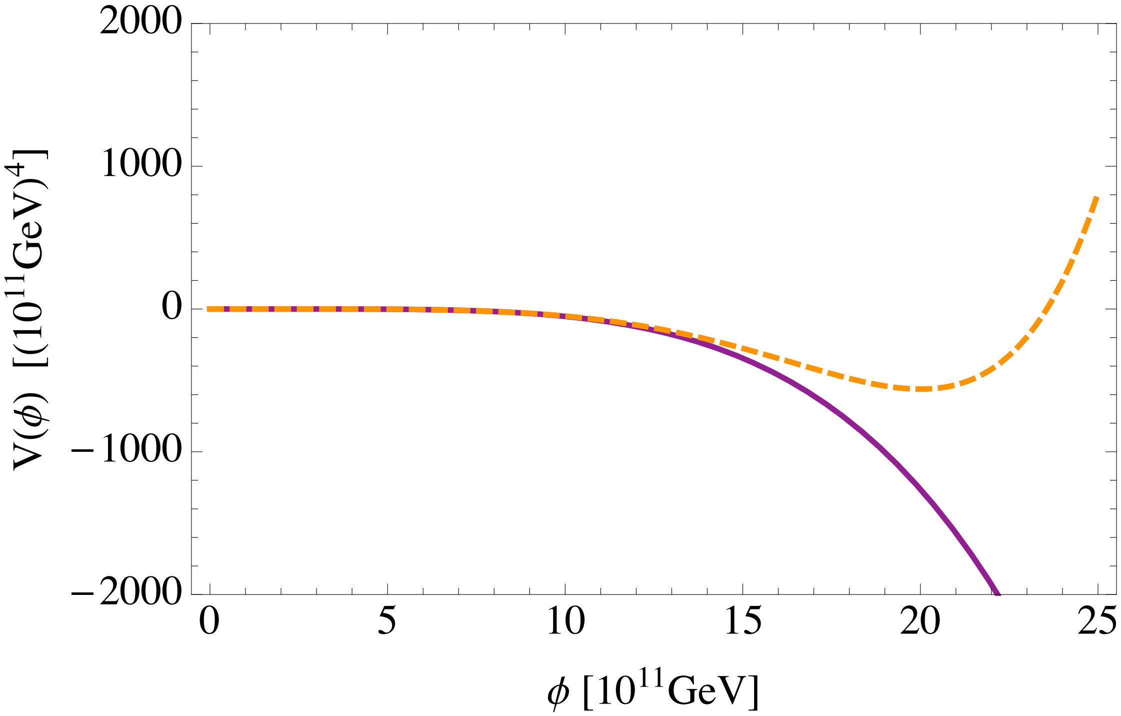
Necessary condition for counting particles: field should be nearly free at t=0.

At the end of the collapse,
may slide over the saddle point, get pulled down into the neighboring minimum.

For natural values of r_0 (so that the evolution is not over many decades of field strength or time), can perform simple numerical simulations.

⇒ generate a "phase diagram" showing critical initial conditions

To simplify numerics, consider a generic stabilization of the potential:

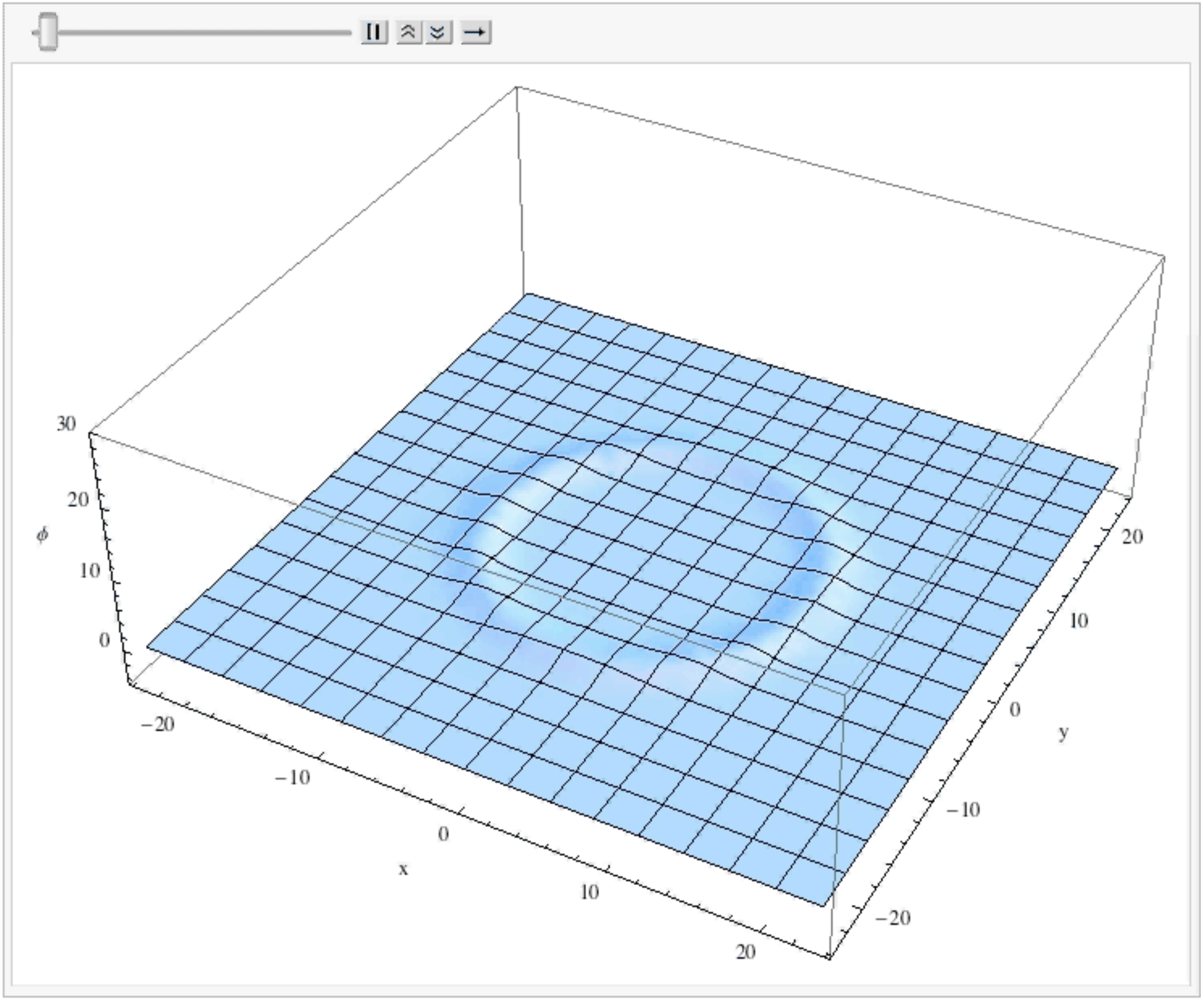


$$V(\phi) \approx 0.0385|\phi|^{3.85} - 0.0324\phi^4 + \lambda_6\phi^6 + \lambda_8\phi^8 \,,$$

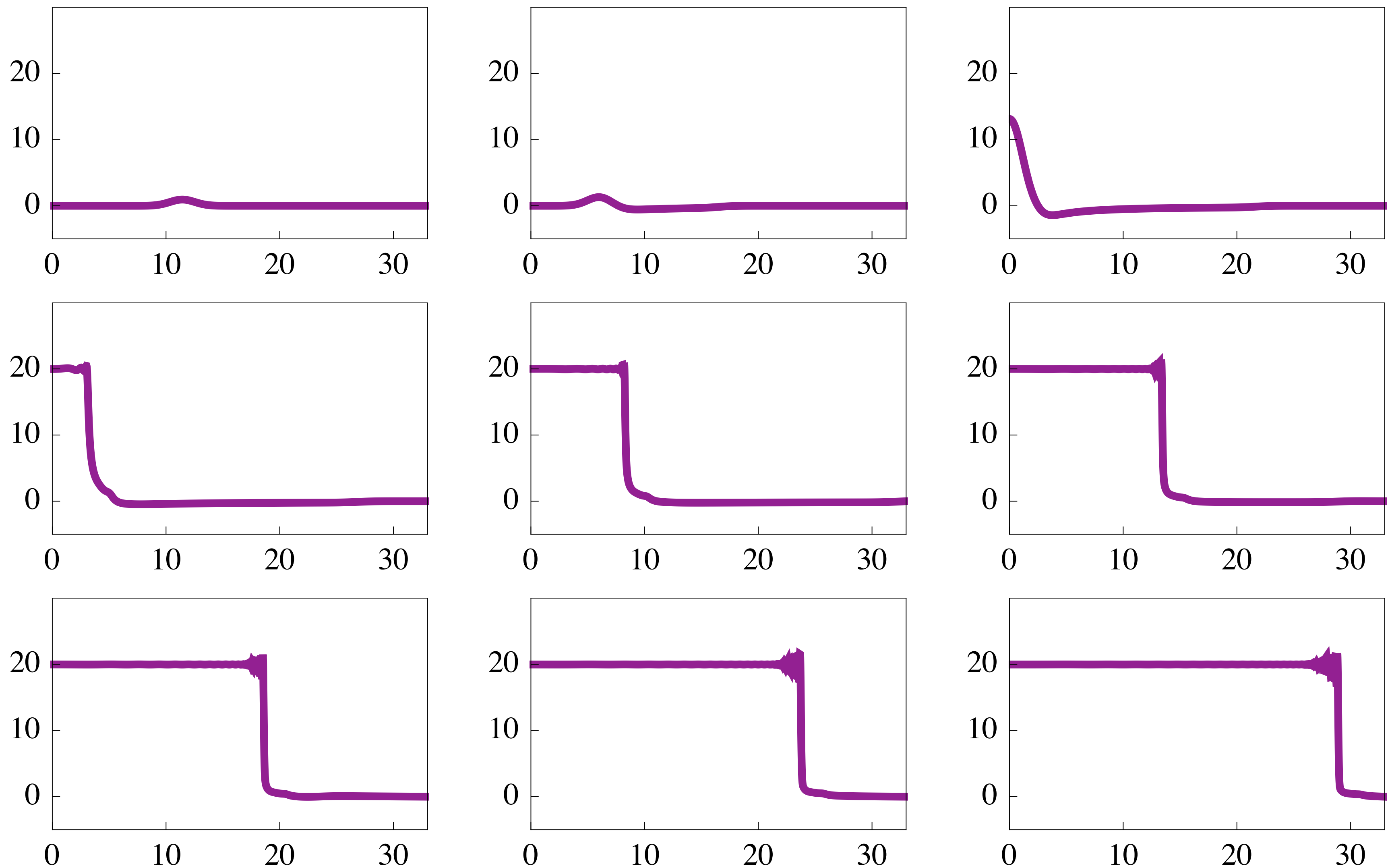
$$\lambda_6 = -2.18 \times 10^{-7} \,, \quad \lambda_8 = 2.79 \times 10^{-8} \,.$$

units of $(10^{11}\ \mathrm{GeV})^4$

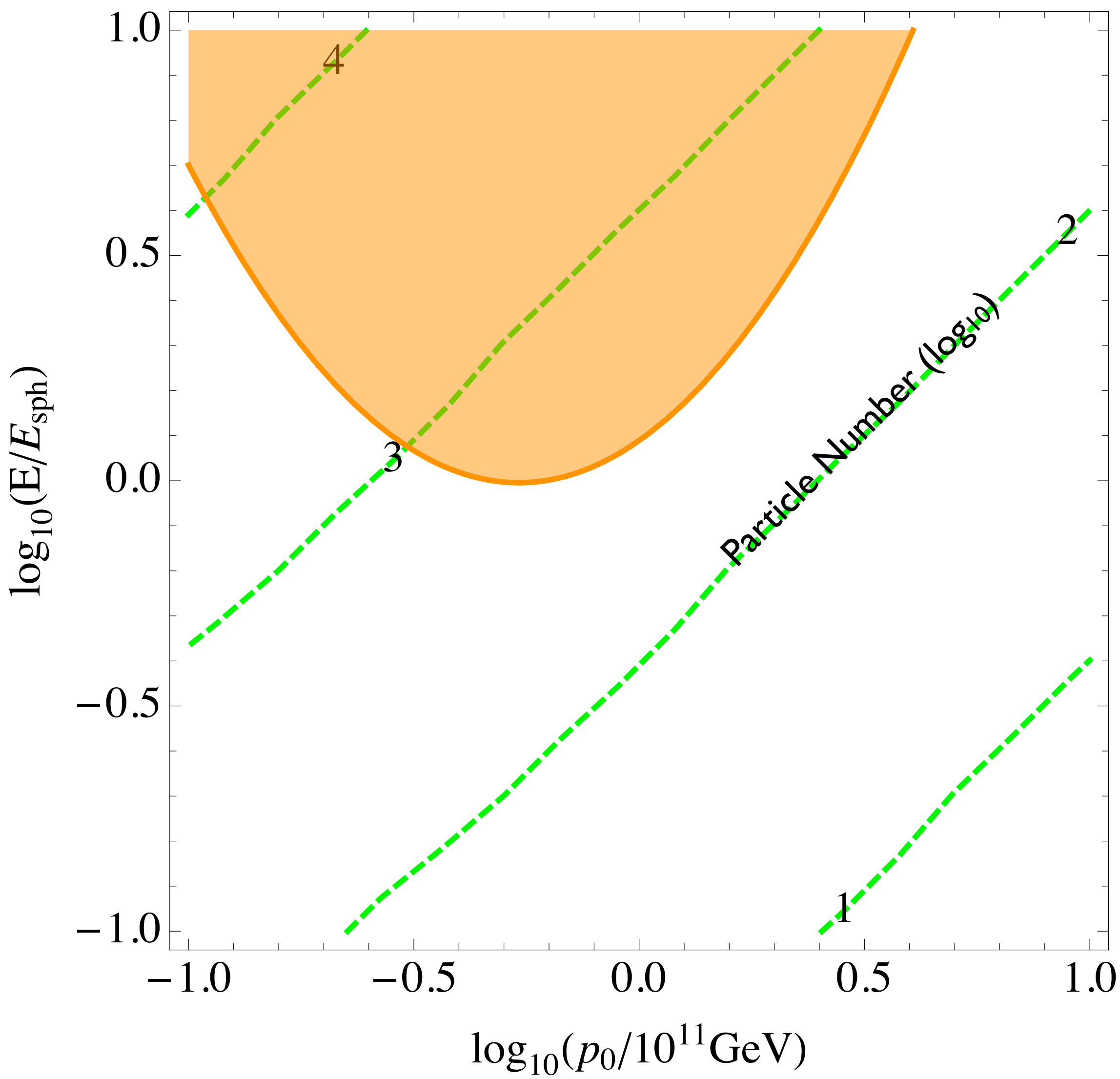
Example Simulation



Example Simulation



Phase Diagram



We have been talking about particle numbers for a reason.

Is there an initial Fock state with large coherent-state overlap?

Pretty clear this can't happen in pair collision.

"The point is that, although the system has sufficient kinetic energy to climb the barrier, it corresponds to motion in the wrong direction in configuration space. It is as though we tried to kick a ball over a hill by kicking it in the wrong direction."

Singleton, Susskind, Thorlacius (1990), arguing against
the possibility of rapid baryon-number violation at the SSC.

There is an (N,p) Fock state that does not have exponentially suppressed overlap with the coherent state.

If p is chosen optimally,

$$\ln \langle F|f\rangle \simeq -\frac{4\pi}{2}\ln((N/4\pi)!) - \frac{\bar{N}}{2} + \frac{N}{2}\ln(\bar{N}/4\pi)$$

Exponential suppression goes away for $N \sim \bar{N}$.

Naively, looks like ~300 Higgs bosons @ 10^{11} GeV

(10^{11} GeV ILC would stretch from the Sun to Saturn)

However,

(1) the N Higgs bosons are localized in a volume of order p^{-3}.
p^{-1} is a billion times smaller than the Compton wavelength!

Have to increase particle number by a billion billion to allow transverse delocalization

(2) how to do you simultaneously make N>1 Higgs bosons?

Summary of part two

In theory we know how to intentionally destabilize the Higgs vacuum,
but it is probably impossible to achieve in practice.

Upside: No serious danger of nihilistic aliens destroying the universe.

Downside: Probably no easy way to mine energy from the true vacuum, either, e.g., stabilizing bubbles of true vacuum
(although if you could you would still be risking cataclysm)

Thanks!